5.3 Energy Transformations

The law of conservation of energy is one of the most useful tools in physics. Since work and energy are scalar quantities, directions are not involved, as they are in momentum. As a result, vector diagrams are not needed, and angles do not have to be calculated. In any given event, the problem is usually to identify the types of energy involved and to ensure that the total energy in all its different forms at the end of the event equals the total at the beginning.

The analysis is often easiest when the motion occurs in a horizontal plane. No change in gravitational potential energy is involved. The following sample problem illustrates this feature.

SECTION EXPECTATIONS

- Analyze situations involving the concepts of mechanical energy, thermal energy, and its transfer.
- Analyze situations involving the concept of conservation of energy.

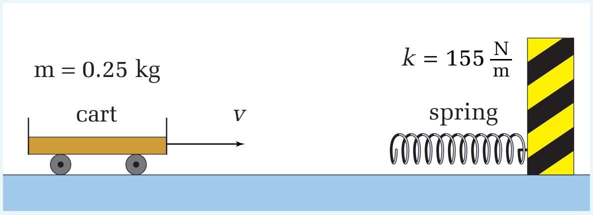
KEY TERMS

- conservative force
- non-conservative force

SAMPLE PROBLEM

Horizontal Elastic Collisions

A low-friction cart with a mass of 0.25 kg travels along a horizontal track and collides head on with a spring that has a spring constant of 155 N/m. If the spring was compressed by 6.0 cm, how fast was the cart initially travelling?



Conceptualize the Problem

- The cart is *moving* so it has *kinetic energy*.
- The spring does *negative work* on the cart, lowering its *kinetic energy*.
- The cart does *work* on the spring, giving it *elastic potential energy*.
- The *height* of the cart does *not change*, so there is no change in *gravitational potential energy*.
- The term *low friction* tells you to neglect the energy lost to work done by friction.
- The law of conservation of energy applies to this problem.

Identify the Goal

The initial speed, *v*, of the cart

Identify the Variables and Constants

Known		Unknown
$m = 0.25$ kg	$x = 0.060$ m	v
$k = 155\ \frac{N}{m}$		

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Develop a Strategy

Write the law of conservation of energy, including the energy quantities associated with the interaction.

$$E'_k + E'_e = E_k + E_e$$

Initially, the spring was not compressed, so the initial elastic potential energy was zero.

$$E_e = 0 \text{ J}$$

After the interaction, the cart stopped, so the kinetic energy was zero.

$$E'_k = 0 \text{ J}$$

Substitute the values for energy listed above.

$$0 \text{ J} + E'_e = E_k + 0 \text{ J}$$
$$E_k = E'_e$$

Expand by substituting the expressions for the energies.

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

Solve for the initial velocity.

$$v = \sqrt{\frac{kx^2}{m}}$$

Substitute numerical values and solve.

$$v = \sqrt{\frac{\left(155\ \frac{\text{N}}{\text{m}}\right)(0.060 \text{ m})^2}{0.25 \text{ kg}}}$$
$$v = 1.493\ 99\ \frac{\text{m}}{\text{s}}$$
$$v \cong 1.5\ \frac{\text{m}}{\text{s}}$$

The cart was travelling at approximately 1.5 m/s before the collision.

Validate the Solution

Unit analysis of the equation $v = \sqrt{\frac{kx^2}{m}}$ shows that it is equivalent to m/s, the standard units for velocity. $\sqrt{\frac{\frac{\text{N}}{\cancel{\text{m}}}\text{m}^{\cancel{2}}}{\text{kg}}} = \sqrt{\frac{\frac{\cancel{\text{kg}}\cdot\text{m}}{\text{s}^2}\text{m}}{\cancel{\text{kg}}}} = \sqrt{\frac{\text{m}^2}{\text{s}^2}} = \frac{\text{m}}{\text{s}}$.

A velocity of 1.5 m/s is reasonable for a lab cart.

PRACTICE PROBLEMS

22. A 1.2 kg dynamics cart is rolling to the right along a horizontal lab desk at 3.6 m/s, when it collides head on with a spring bumper that has a spring constant of 2.00×10^2 N/m.

(a) Determine the maximum compression of the spring.

(b) Determine the speed of the cart at the moment that the spring was compressed by 0.10 m.

(c) Determine the acceleration of the cart at the moment that the spring was compressed 0.10 m.

23. A circus car with a clown has a total mass of 150 kg. It is coasting at 6.0 m/s, when it hits a large spring head on. If it is brought to a stop by the time the spring is compressed 2.0 m, what is the spring constant of the spring?

The analysis becomes a bit more complicated when the motion is vertical, since there are now changes in gravitational potential energy along with elastic potential energy and kinetic energy.

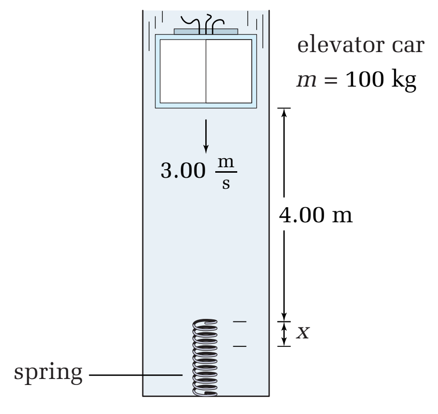
PHYSICS FILE

The energies discussed here are commonly found in mechanical systems with springs and pulleys. As a result, kinetic energy, gravitational potential energy, and elastic potential energy are commonly referred to as "mechanical energy."

SAMPLE PROBLEM

Vertical Elastic Collisions

A freight elevator car with a total mass of 100.0 kg is moving downward at 3.00 m/s, when the cable snaps. The car falls 4.00 m onto a huge spring with a spring constant of 8.000×10^3 N/m. By how much will the spring be compressed when the car reaches zero velocity?



Conceptualize the Problem

- Initially, the car is in *motion* and therefore has *kinetic energy*. It also has *gravitational potential energy*.
- As the car begins to *fall*, the *gravitational potential energy* transforms into *kinetic energy*. When the elevator hits the spring, the elevator *slows*, losing *kinetic energy*, and the spring compresses, gaining *elastic potential energy*.
- When the elevator comes to a complete *stop*, it has *no kinetic* or *gravitational potential energy*. All of the energy is now stored in the spring in the form of *elastic potential energy*.
- Since all of the motion is in a downward direction, define "down" as the positive direction for this problem.

Identify the Goal

The compression of the spring, x, when the car comes to rest

Identify the Variables and Constants

Known	Implied	Unknown
$m_{\text{car}} = 100.0$ kg	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	x
$v = 3.00\ \frac{\text{m}}{\text{s}}$ [down]		
$k = 8.000 \times 10^3\ \frac{\text{N}}{\text{m}}$		
$h_{(\text{above spring})} = 4.00$ m		

Develop a Strategy

Write the law of conservation of energy for the forms of energy involved in the problem.

$$E'_{\text{g}} + E'_{\text{e}} + E'_{\text{k}} = E_{\text{g}} + E_{\text{e}} + E_{\text{k}}$$

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Choose the lowest level of the elevator (maximum compression of the spring) as the reference level for gravitational potential energy.

$E'_g = 0\ \mathrm{J}$

The car comes to a rest at the lowest point.

$E'_k = 0\ \mathrm{J}$

Initially, the spring is not compressed.

$E_e = 0\ \mathrm{J}$

Substitute these initial and final conditions into the equation for conservation of energy and simplify.

$$0\ \mathrm{J} + E'_e + 0\ \mathrm{J} = E_g + 0\ \mathrm{J} + E_k$$
$$E'_e = E_g + E_k$$

Expand by substituting the expressions for the various forms of energy.

$$\frac{1}{2}kx^2 = mg\Delta h + \frac{1}{2}mv^2$$

The change in height for the gravitational potential is 4.00 m, plus the compression of the spring, x. Substitute this expression into the equation.

$$\frac{1}{2}kx^2 = mg(4.00 + x) + \frac{1}{2}mv^2$$

Since the equation yields a quadratic equation, you cannot solve for x. Substitute in the numerical values and rearrange so that the right-hand side is zero.

$$\frac{1}{2}\left(8.00 \times 10^3\ \frac{\mathrm{N}}{\mathrm{m}}\right)x^2 = (100.0\ \mathrm{kg})\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)(4.00 + x)\ \mathrm{m} + \frac{1}{2}(100.0\ \mathrm{kg})\left(3.00\ \frac{\mathrm{m}}{\mathrm{s}}\right)^2$$

$$4.00 \times 10^3x^2 - 981x - 3924 - 450 = 0$$

$$4.00 \times 10^3x^2 - 981x - 4374 = 0$$

Use the quadratic formula to find the value of x.

$$x = \frac{981 \pm \sqrt{(-981)^2 - 4(4.00 \times 10^3)(-4374)}}{2(4.00 \times 10^3)}$$

$$x = 1.1756\ \mathrm{m} \quad \text{or} \quad -0.930\ 25\ \mathrm{m}$$

$$x \cong 1.18\ \mathrm{m}$$

Compression cannot be negative (or the spring would be stretching), so choose the positive value. The spring was compressed 1.18 m.

Validate the Solution

The units on the left-hand side of the final equation are $\frac{\mathrm{N}}{\mathrm{m}} \cdot \mathrm{m}^2 = \frac{\frac{\mathrm{kg \cdot m}}{\mathrm{s}^2}}{\mathrm{m}} \cdot \mathrm{m}^2 = \frac{\mathrm{kg \cdot m^2}}{\mathrm{s}^2}$.

On the right-hand side of the equation, the units are $\mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}^2} \cdot \mathrm{m} = \frac{\mathrm{kg \cdot m^2}}{\mathrm{s}^2}$.

Both sides of the equation have the same units, so you can have confidence in the equation. The answer is also in a range that would be expected with actual springs.

Note: The negative root in this problem is interesting in that it does have meaning. If the car had somehow latched onto the spring during the collision, the negative value would represent the maximum extension of the spring if the car had bounced up from the bottom due to the upward push of the spring.

PRACTICE PROBLEMS

24. A 70.0 kg person steps through the window of a burning building and drops to a rescue net held 8.00 m below. If the surface of the net is 1.40 m above the ground, what must be the value of the spring constant for the net so that the person just touches the ground when the net stretches downward?

25. A 6.0 kg block is falling toward a spring located 1.80 m below. If it has a speed of 4.0 m/s at that instant, what will be the maximum compression of the spring? The spring constant is 2.000×10^3 N/m.

26. In a "head dip" bungee jump from a bridge over a river, the bungee cord is fastened to the jumper's ankles. The jumper then steps off and falls toward the river until the cord becomes taut. At that point, the cord begins to slow the jumper's descent, until his head just touches the water. The bridge is 22.0 m above the river. The unstretched length of the cord is 12.2 m. The jumper is 1.80 m tall and has a mass of 60.0 kg. Determine the

(a) required value of the spring constant for this jump to be successful

(b) acceleration of the jumper at the bottom of the descent

Conservative and Non-Conservative Forces

Until now, you have been asked to assume that objects could move without friction. A pendulum would keep swinging repeatedly with the same amplitude, continuously converting energy between kinetic and gravitational potential forms of energy. A skier could slide down a hill, converting gravitational potential energy into kinetic energy and then, faced with an upward slope, could keep on going, converting the kinetic energy back into potential energy until the original height was reached.

The forces with which you have been dealing are referred to as **conservative forces**. This means that the amount of work that they do on a moving object does not depend on the path taken by that object. In the absence of friction, the boulder in Figure 5.15 will reach the bottom of the hill with the same kinetic energy and speed whether it dropped off the cliff on the left or slid down the slope on the right.

COURSE CHALLENGE

Energy Transformations

Light energy is transformed into stored chemical energy each time you take a photograph. The operation of infrared cameras, ultrasound images, and video cameras also relies on various energy transformations. Refer to page 604 for suggestions on relating energy transformations to your *Course Challenge*.

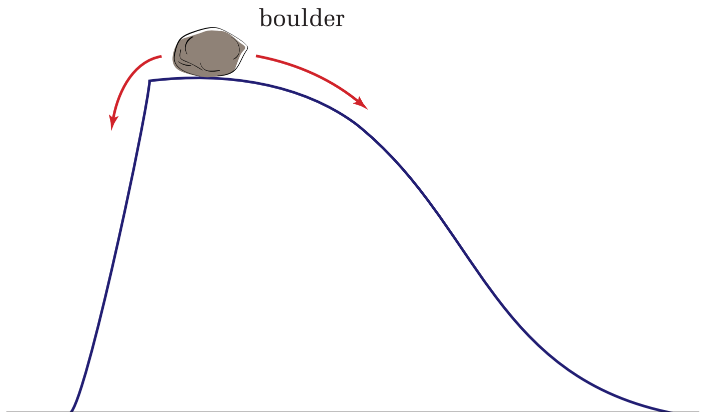


Figure 5.15 Gravity is a conservative force. If the boulder was dropped over the edge of the cliff, all of the gravitational potential energy would be converted into kinetic energy. Friction is not a conservative force. If the boulder slides down the hill, the kinetic energy at the bottom will not be as great as it would if the boulder fell straight down.

ELECTRONIC LEARNING PARTNER

To enhance your understanding of energy transformation, refer to your Electronic Learning Partner.

Friction is a **non-conservative force**. The amount of work done by a non-conservative force depends on the path taken by the force and the object. For example, the amount of energy transferred to the snow in Figure 5.16 depends on the path taken by the skier. The skier going straight down the slope should reach the bottom with a greater speed than the skier who is tracking back and forth across the slope.

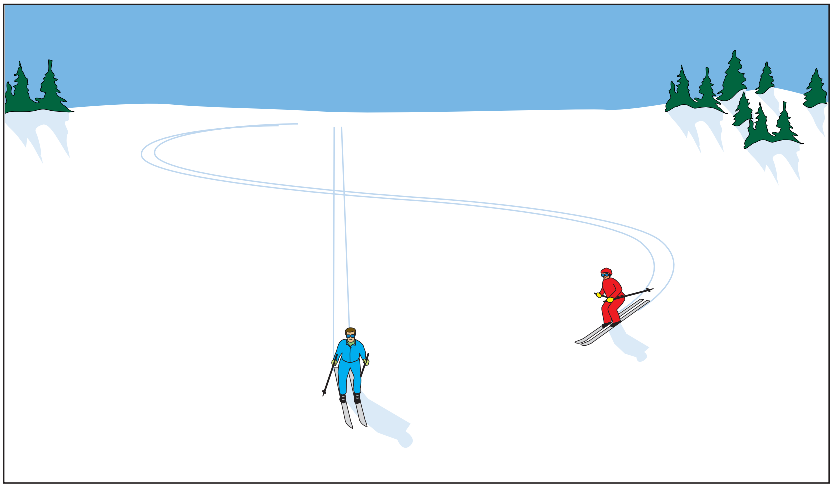


Figure 5.16 Although friction between the skis and the snow is small, friction nevertheless does some work on the skiers, slowing their velocity a little. The work done by friction is greater along the longer of the paths.

Friction causes the skier to do work on the environment. The snow heats up slightly and is moved around. For the skier, this is negative work — the skier is losing energy and cannot regain it as useful kinetic or potential energy. The sum of the skier's kinetic and gravitational potential energy at the end of the run will be less than it was at the start.

Wind pressure is another example of a non-conservative force. If the skier had the wind coming from behind, the wind (the environment) could be doing work on the skier. This would be positive work. The sum of the skier's kinetic and gravitational potential energies could increase beyond the initial total. However, the amount of energy transferred by the wind would depend to a large extent on the path of the skier, so the wind would be a non-conservative force.

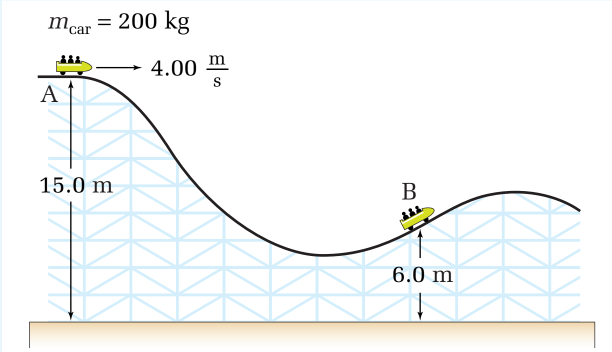
When dealing with non-conservative forces, the law of conservation of energy still applies. However, you must account for the energy exchanged between the moving object and its environment. One approach to this type of situation is to define the system as the skier and the local environment; that is, the skier, wind, and snow become the system. The following sample problem illustrates this concept.

PHYSICS FILE

Quite often, you might not want your forces to be conservative. Without friction, many of your clothes would simply fall apart into strands as you moved. In addition, keep-fit programs would have to be greatly modified. A person who rides an exercise bicycle to lose mass (through chemical reactions that provide the energy) does not want the energy back. It simply is dissipated as sound and heat. Likewise, the weight lifter who does work to lift a bar bell does not expect to receive that energy back when the bar bell is lowered.

SAMPLE PROBLEM

Energy Conversions on a Roller Coaster



A roller-coaster car with a mass of 200.0 kg (including the riders) is moving to the right at a speed of 4.00 m/s at point A in the diagram. This point is 15.00 m above the ground. The car then heads down the slope toward point B, which is 6.00 m above the ground. If 3.40×10^3 J of heat energy are produced through friction between points A and B, determine the speed of the car at point B.

Conceptualize the Problem

- As the roller-coaster car *moves* down the track, most of the *gravitational potential energy* is converted into *kinetic energy*, but some is lost as *heat* due to *friction*.
- The law of conservation of total energy applies.
- *Heat energy* must be included as a *final energy*.

Identify the Goal

The speed of the car at point B, v_B

Identify the Variables and Constants

Known		Implied	Unknown
$h_A = 15.00$ m	$E_{heat} = 3.40 \times 10^3$ J	$g = 9.81\ \frac{m}{s^2}$	v_B
$h_B = 6.00$ m	$m = 200.0$ kg		
$v_A = 4.00\ \frac{m}{s}$			

Develop a Strategy

Write the law of conservation of energy, including heat as a final energy form.

$$E_k' + E_g' + E_{heat} = E_k + E_g$$

Expand by substituting the expressions for the forms of energy. Solve for the speed of the car at point B

$$\frac{1}{2}mv_B^2 + mgh_B + E_{heat} = \frac{1}{2}mv_A^2 + mgh_A$$

$$\frac{1}{2}mv_B^2 = -mgh_B - E_{heat} + \frac{1}{2}mv_A^2 + mgh_A$$

$$v_B^2 = \frac{2(-mgh_B - E_{heat} + \frac{1}{2}mv_A^2 + mgh_A)}{m}$$

$$v_B = \sqrt{\frac{2(-mgh_B - E_{heat} + \frac{1}{2}m(v_A)^2 + mgh_A)}{m}}$$

$$v_B = \sqrt{\frac{2\left[-\left(1.1772 \times 10^4\ \frac{kg \cdot m^2}{s^2}\right) - (3.40 \times 10^3\ J) + \left(1.6 \times 10^3\ kg\frac{m^2}{s^2}\right) + \left(2.943 \times 10^4\ \frac{kg \cdot m^2}{s^2}\right)\right]}{200.0\ kg}}$$

$$v_B = \sqrt{\frac{3.1716 \times 10^4\ \frac{\cancel{kg} \cdot m^2}{s^2}}{200.0\ \cancel{kg}}}$$

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$$v_B = \sqrt{1.5858 \times 10^2 \ \frac{m^2}{s^2}}$$

$$v_B = 1.2593 \times 10^1 \ \frac{m}{s}$$

$$v_B \cong 12.6 \ \frac{m}{s}$$

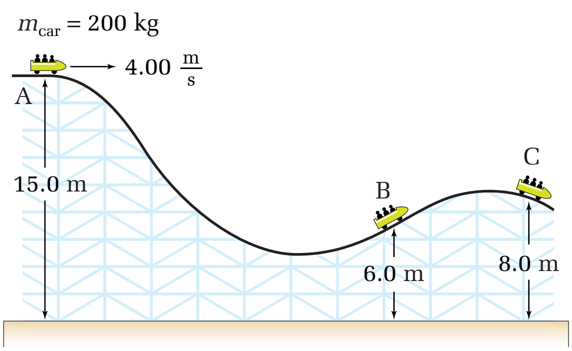
The speed of the car at point B will be 12.6 m/s.

Validate the Solution

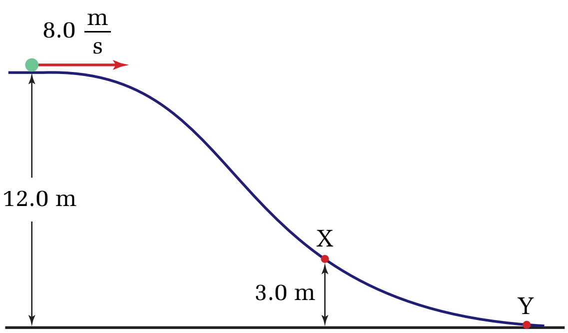
The speed at point B is expected to be larger than its speed at point A.

PRACTICE PROBLEMS

27. Determine the speed of the roller-coaster car in the sample problem at point C if point C is 8.0 m above the ground and another 4.00×10^2 J of heat energy are dissipated by friction between points B and C.



28. A sled at the top of a snowy hill is moving forward at 8.0 m/s, as shown in the diagram. The height of the hill is 12.0 m. The total mass of the sled and rider is 70.0 kg. Determine the speed of the sled at point X, which is 3.0 m above the base of the hill, if the sled does 1.22×10^3 J of work on the snow on the way to point X.



29. If the sled in the previous question reaches the base of the hill with a speed of 15.6 m/s, how much work was done by the snow on the sled between points X and Y?

PROBEWARE

If your school has probeware equipment, visit ***www.mcgrawhill.ca/links/physics12*** and follow the links for an in-depth activity on energy, Hooke's law, and simple harmonic motion.

In solving these problems, you have assumed that the value for the acceleration due to gravity (g) is constant at 9.81 m/s^2. You probably recall reading that this value is valid only for a small region close to Earth's surface. In Chapter 3, you learned that, as you go to the higher altitudes, the acceleration due to gravity decreases. You worked with forces of gravity at any distance from Earth, other planets, and even stars. You learned how to calculate the radii of orbits and orbital speed of satellites.

In the next chapter, you will focus on the energy requirements for sending a satellite into orbit and even for escaping Earth's gravitational pull entirely. You will also learn the importance of the conservation of momentum in navigating through space.